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CS200

Homework #21

1.

We will prove that the maximum number of vertices at depth is in a binary tree is

**Base case: d = 0:** . At depth 0, the maximum number of vertices must be 1 because there will only be the root vertex which has a depth of 0. Therefore, the base case holds.

**Inductive hypothesis: d > 0 or d+1:** = . At depth , we have already proven the base case of and when depth increases by 1, the maximum number of vertices must be double the last layer which is resembled by . Therefore, the recursive case holds.

**Conclusion:** Because we have proven that the base case and the recursive case, we have proven that the maximum number of vertices at depth is in a binary tree is .

2.

The maximum number of vertices at height is because the maximum number of vertices of height is the same number of vertices as the maximum number of vertices at depth but you have to multiply by 2 for an extra layer and minus the root node (as shown by my equivalence expression above).

3.

Round down the equation:

4.

The asymptotic height of a binary tree with n vertices can be expressed as O(logn).